4.8: Antiderivatives

Def: A function F is an antiderivative of f if $F'(x) = f(x)$.

Remark: (1) The antiderivative is the "opposite" of the derivative.

(2) Notice an instead of the in definition implies there are many antiderivatives.

Ex(1): ~~Let~~ Find ~~the~~ an antiderivative.

(a) $f(x) = 2x$ (b) $g(x) = \cos x$ (c) $h(x) = \frac{1}{x} + 2e^{2x}$

$F(x) = x^2$ $G(x) = \sin x$ $H(x) = \ln x + e^{2x}$.

Theorem: If F is an antiderivative of f then $F(x) + C$ is also an antiderivative for an arbitrary constant C.

$\{F(x) + C : C \in \mathbb{R}\}$ is the family of antiderivatives of f.

Ex (2): Find the antiderivative of $f(x) = 3x^2$ such that $F(1) = -1$.

$F(x) = x^3 + C$. $F(1) = 1^3 + C = -1$ so $C = -2$

Thus $F(x) = x^3 - 2$.

	Function	General Antiderivative
(1)	x^n	$\frac{x^{n+1}}{n+1} + C$
(2)	$\sin x$	$-\cos x + C$
(3)	$\cos x$	$\sin x + C$
(4)	$\sec^2 x$	$\tan x + C$
(5)	$\csc^2 x$	$-\cot x + C$
(6)	$\sec x \tan x$	$\sec x + C$
(7)	$\csc x \cot x$	$-\csc x + C$
(8)	e^x	$e^x + C$
(9)	$\frac{1}{x}$	$\ln\lvert x\rvert + C$
(10)	a^x	$\frac{a^x}{\ln a} + C$
(11)	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
(12)	$\frac{1}{1+x^2}$	$\tan^{-1} x + C$
(13)	[illegible]	$\sec^{-1} x + C$

Rules:

		Function	Antiderivative
$K \in \mathbb{R}$	(1)	$Kf(x)$	$KF(x)+C$
	(2)	$f(x) \pm g(x)$	$F(x) \pm G(x)$

Ex(3): Find the general antiderivative.

(a) $x^5 \Rightarrow \frac{x^6}{6}+C$

(b) $\frac{1}{\sqrt{x}} \Rightarrow 2\sqrt{x}+C$

(c) $\sin(2x) \Rightarrow -\frac{\cos(2x)}{2}+C$

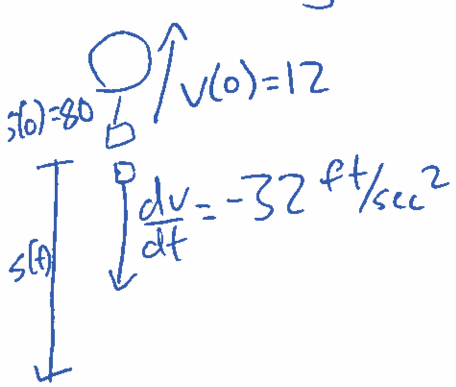
(d) $\cos\left(\frac{x}{2}\right) \Rightarrow 2\sin\left(\frac{x}{2}\right)+C$

(e) $e^{-3x} \Rightarrow -\frac{e^{-3x}}{3}+C$

(f) $2^x \Rightarrow \frac{2^x}{\ln 2}+C$

Differential Eqns and Initial Value Problems

Ex(4): A Hot-air balloon is ascending at a rate of 12 ft/sec. and drops a package 80 ft above ground. How long will it take the package to reach the ground



$\frac{dv}{dt}=-32$ w/ $v(0)=12$ (initial value)

$v(t)=-32t+C$ so $C=12$

$v(t)=-32t+12$

$s(t)=-16t^2+12t+C \Rightarrow C=80$

So $s(t)=-16t^2+12t+80 \Rightarrow t=2.64$ sec.

Indefinite Integrals:

The family of antiderivatives of f is called the <u>indefinite integral</u> of f with respect to x, denoted

$$\underbrace{\int}_{\text{integral sign}} \underbrace{f(x)}_{\text{integrand}}\, d\underbrace{x}_{\text{variable of integration}} = F(x) + C.$$

Ex(s): Find the desired family.

(a) $\int 2x\,dx = x^2 + C$

(b) $\int \frac{1}{2x} + e^{3x}\,dx = \frac{\ln x}{2} + \frac{e^{3x}}{3} + C$

(c) $\int (x^2 - 2x + 5)\,dx = \frac{x^3}{3} - x^2 + 5x + C$

(d) $\int \cos t\,dt = \sin t + C$